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Action graph of a semigroup act & its functorial connection

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Abstract. In this paper we define *C-induced action graph* $G(S, a, C; A)$ corresponding to a semigroup act (S, a, A) and a subset C of S. This generalizes many interesting graphs including *Cayley Graph* of groups and semigroups, *Transformation Graphs (TRAG)*, *Group Action Graphs (GAG)*, *Derangement Action Graphs*, *Directed Power Graphs of Semigroups* etc. We focus on the case when $C = S$ and name the digraph, so obtained, as *Action Graph of a Semigroup Act* (S, a, A). Some basic structural properties of this graph follow from algebraic properties of the underlying semigroup and its action on the set. Action graph of a strongly faithful act is also studied and graph theoretic characterization of a strongly faithful semigroup act as well as that of idempotents in a semigroup are obtained. We introduce the notion of *strongly transitive digraphs* and based on this we characterize action graphs of semigroup acts in the class of simple digraphs. The simple fact that morphism between semigroup acts leads to digraph homomorphism between corresponding action graphs, motivates us to represent action graph construction as a functor from the category of semigroup acts to the category of certain digraphs. We capture its functorial properties, some of which

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